MECHANICS (C) UNIT 1 **TEST PAPER 9**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A particle P moves in a straight line with acceleration $t - \frac{4}{t^2}$ ms^{-2} at time t seconds, for $t \geq 1$. When $t = 2$, P is instantaneously at rest.
   Calculate the magnitude of the velocity of P when $t = 4$. [5]

2. A book rests on a rough desk-lid which is hinged at one end. When the lid is raised so that it makes an angle of 15° with the horizontal, the book is just on the point of sliding down a line of greatest slope. Modelling the book as a particle, find
   (i) the coefficient of friction between the book and the desk-lid, [2]
   (ii) the acceleration with which the book starts to move if it is released from rest when the lid is inclined at 20° to the horizontal. [4]

3. A jet of water issues from a cylindrical pipe with a circular cross-section of radius 2·75 cm. The water strikes a vertical wall at a speed of 9 ms^{-1}.
   Taking the density of water to be 1000 kg m^{-3}, calculate the momentum destroyed each second by the impact with the wall. State one modelling assumption that you have made. [6]

4. A particle P is projected vertically upwards from ground level at time $t = 0$ with speed 20 ms^{-1}. Two seconds later another particle Q is projected vertically upwards with speed 30 ms^{-1} from a point on the same horizontal ground.
   (i) Taking the upward direction as positive, write down expressions in terms of g and t for the velocities of P and of Q at time t seconds after P is projected. [3]
   (ii) Find the value of t when both particles are moving with the same speed. [4]

5. Two particles A and B, of mass 1 kg and m kg respectively, where $m > 1$, are attached to the ends of a light inextensible string which passes over a small fixed smooth pulley.
   The particles are released from rest and move with the string taut and vertical.
   (i) Show that the acceleration of the system is equal to $\frac{(m-1)g}{m+1}$. [6]
   (ii) Find the tension in the string, in terms of m and g, expressing your answer as a single algebraic fraction in its simplest form. [2]

   When the system is released from rest, both particles are 52·5 cm above ground level and 60 cm below the level of the pulley. B hits the ground after half a second.
   (iii) Find the value of m. [5]
   (iv) Find the speed with which B hits the ground. [3]

6. A particle starts from rest and accelerates at a uniform rate over a distance of 12 m. It then travels at a constant speed of u ms^{-1} for a further 30 seconds. Finally it decelerates uniformly to rest at 1·6 ms^{-2}.

   (i) Sketch the velocity-time graph for this motion. [3]

   (ii) Show that the total time for which the particle is in motion is

   $$\frac{5u}{8} + 30 + \frac{24}{u} \text{ seconds.}$$ [5]

   (iii) Find, in terms of u, the total distance travelled by the particle during the motion. [3]

   (iv) Given that the total time for the motion is 39·5 seconds, show that $5u^2 - 76u + 192 = 0$. [3]

   (v) Find the two possible values of u and the total distance travelled in each case. [6]

MECHANICS 1 (C) TEST PAPER 9 : ANSWERS AND MARK SCHEME

1. $v = \int a\,dt = \frac{1}{2}t^2 + \frac{4}{t} + c$ $\quad v(2) = 0 : c = -4$ M1 A1 A1

   $t = 4 :\ v = 8 + 1 - 4 = 5\ \text{ms}^{-1}$ M1 A1 5

2. (i) $R = W\cos 15°,\ \mu R = W\sin 15°$ $\quad \mu = \tan 15° = 0{\cdot}268$ M1 A1

   (ii) Acc down plane $= g\sin 20° - \mu g\cos 20° = 0{\cdot}0902g = 0{\cdot}884\ \text{ms}^{-2}$ M1 A1 M1 A1 6

3. Volume per second $= \pi(0{\cdot}0275)^2 \times 9 = 0{\cdot}02138\ \text{m}^3$ M1 A1

   having mass 21·38 kg $\quad$ Momentum $= mv = 21{\cdot}38 \times 9 = 192$ Ns A1 M1 A1

   Assumed water moves horizontally, does not rebound, etc. B1 6

4. (i) $v_P = 20 - 9{\cdot}8t$ $\quad v_Q = 30 - 9{\cdot}8(t-2)$ B1 M1 A1

   (ii) Equal speeds when $v_P = v_Q$ (never) or $v_P = -v_Q$: M1

   $20 - 9{\cdot}8t = 9{\cdot}8t - 19{\cdot}6 - 30$ $\quad 19{\cdot}6t = 69{\cdot}6$ $\quad t = 3{\cdot}55$ M1 A1 A1 7

5. (i) $mg - T = ma,\ T - g = a$ $\quad$ Add : $mg - g = ma + a$ M1 A1 A1 M1

   $g(m-1) = a(m+1)$ $\quad a = \frac{(m-1)g}{m+1}$ A1 A1

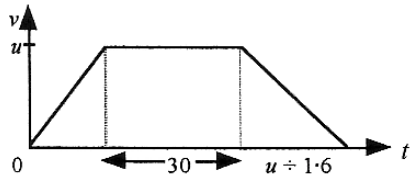
   (ii) $T = a + g = \frac{2mg}{m+1}$ M1 A1

   (iii) $0{\cdot}525 = \frac{1}{2}\frac{(m-1)g}{m+1}(\frac{1}{2})^2$ $\quad \frac{m-1}{m+1} = \frac{3}{7}$ $\quad 7m - 7 = 3m + 3$ M1 A1 A1

   $4m = 10$ $\quad m = 2{\cdot}5$ M1 A1

   (iv) Then $a = \frac{3g}{7} = 4{\cdot}2$ $\quad v = at = 0{\cdot}5a = 2{\cdot}1\ \text{ms}^{-1}$ B1 M1 A1 16

6. (i)



   B3

   (ii) Let three times be t_1, t_2, t_3 $\quad \frac{1}{2}ut_1 = 12$ $\quad t_1 = \frac{24}{u}$ M1 A1

   $t_2 = 30$ $\quad u \div t_3 = 1{\cdot}6$ $\quad t_3 = u \div 1{\cdot}6 = \frac{5u}{8}$ $\quad$ Hence result M1 A1 A1

   (iii) Distance = sum of areas $= 12 + 30u + \frac{1}{2}u\frac{5u}{8} = \frac{5u^2}{16} + 30u + 12$ M1 A1 A1

   (iv) $\frac{5u}{8} + 30 + \frac{24}{u} = 39{\cdot}5$ $\quad \times 8u : 5u^2 + 240u + 192 = 316u$ M1 A1

   $5u^2 - 76u + 192 = 0$ A1

   (v) $(5u - 16)(u - 12) = 0$ $\quad u = 3{\cdot}2$ or $u = 12$ M1 A1 A1

   When $u = 3{\cdot}2$, dist. = 111 m $\quad$ When $u = 12$, dist. = 417 m M1 A1 A1 20